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**Matteoni, Giulia; Georgakis, Christos T.**

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## Pedestrian induced vertical vibrations: Response to running using the Response Spectrum Method

G. Matteoni & C.T. Georgakis

Department of Civil Engineering, Technical University of Denmark, Brovej, Lyngby, Denmark

**ABSTRACT:** Footbridges are becoming increasingly prone to vibrations and designers are generally unable to accurately predict pedestrian-induced vertical vibrations. Many aspects of human loading are in fact not properly accounted for in the load models employed by the international codes of practice, such as the randomness of crowds travelling across the footbridge. Moreover, the codes, for the most part, do not deal with pedestrian loading other than walking, even though running and jumping can often produce larger loads and vibration amplitudes. In this paper, an investigation into the response of footbridges under human-induced loading due to running is undertaken. To this end, Monte Carlo simulations are adopted for the generation of crowd loading. A reference response spectrum, defined as the peak acceleration associated with a given return period, is then developed. Correction factors are adopted to take into account variations in the structural characteristics, crowd morphology and return period. The correction factors, together with the reference acceleration, are used to determine the final response of the footbridge, for a given probability of load occurrence.

**Keywords:** footbridge, vertical vibrations, load model, running, response spectrum.

### 1 INTRODUCTION

Most international codes of practice do not provide an adequate methodology for the response prediction of a footbridge, to the loading produced by a crowd. Moreover the loading due to other kinds of motion, like running or jumping, are often not treated at all.

A codifiable and accurate methodology for the evaluation of footbridge response to running is presented herewith. The methodology is based on the validated Response Spectrum Method, developed and presented by Georgakis & Ingólfsson (2008) for walking crowds. The pedestrian crowd loading is generated through Monte Carlo Simulations and the footbridge response is given in terms of peak acceleration, as a function of the return period of the loading. Since the Response Spectrum is developed for a reference bridge and crowd morphology, correction factors are introduced to accommodate for other structural as well as crowd characteristics.

A pedestrian's behavior when crossing a footbridge is assumed random and therefore the main parameters influencing the loading function, such as pacing frequency and running speed, are modeled according to statistical distributions. The absence of *human-human* and *human-structure* interaction is assumed. Therefore, the crowd consists of  $N_p$  independent pedestrians, each entering the bridge with a constant mean flow. The pedestrian arrival time to the bridge is also assumed random and following a Poisson process.

### 2 MONTE CARLO SIMULATIONS

#### 2.1 Vertical pedestrian load

According to Bachmann and Amman (1987), the vertical component of the physical foot force for running can be represented by the model of *jumping on a spot*, which is mathematically described by a half-sinus wave during the contact time. This model, which reflects the discontinuous nature of the ground reaction force, is adopted in the simulations. For the  $j$ th pedestrian crossing the bridge, the vertical component of the physical footfall force, during the contact interval  $t_c$  is described by equation (1):

$$F_j(t) = k_{p,j} W_j \sin\left(\frac{\pi}{t_{p,j}}(x - v_p(t - t_{ar}))\right) \delta(x - v_p(t - t_{ar})) \quad (1)$$

where  $W_j$  is the self-weight of the  $j$ th pedestrian,  $k_{p,j}$  is the impact factor, and  $t_{p,j}$  is the contact time. During the no-contact interval,  $t_{nc}$ , the physical foot force is set equal to zero.

The loading due to a group of  $N_p$  pedestrians is then calculated as a superimposition of the contributions due to each single pedestrian, so that:

$$F(t) = \sum_{j=1}^{N_p} F_j(t) \quad (2)$$

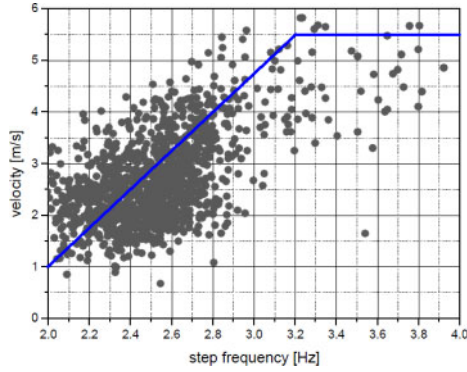


Figure 1. Running velocity vs step frequency and fitted bilinear model. Data points from Kasperski and Sahnaci (2008).

(a) Body weight

In the simulations, a normal distribution is assumed for the body weight of the population crossing the bridge with a mean value and a standard deviation equal to 74.00 kg and 14.72 kg, respectively. These values have been obtained from data published by the Danish Institute of Public Health.

(b) Pacing rate and running speed

Kasperski & Sahanci (2008) observed a positive correlation between the step frequency and the running velocity, with a large scatter reflecting the different body sizes and frequency preferences. The observed step frequencies are in the range of 2 to 4 Hz while the observed velocities are between 1 and 6 m/s. A *bi-linear model* is fitted to the tendencies reported by the authors, so that:

$$v_{p,j} [m/s] = 3.33 f_{p,j} - 5.16 \quad f_{p,j} \leq 3.2 \text{ Hz} \quad (3.a)$$

$$v_{p,j} [m/s] = 5.50 \quad f_{p,j} \geq 3.2 \text{ Hz} \quad (3.b)$$

Figure 1 shows the experimental data collected by Kasperski and Sahanci (2008) during the tests conducted on a sample of 105 volunteers, 88 males and 17 females, during the scenario *short distance running with a specific purpose*, as well as the fitted *bi-linear* fitted model.

Minimum and maximum allowable frequencies are also defined in the model and are equal to 2.0 Hz and 3.5 Hz respectively.

The power law model defined by Ingólfsson et al. (2007), describing the variation of step frequency with running speed cannot be extended over the frequency range of running activities. In fact, for this frequency range and particularly for frequencies higher than 3 Hz the application of the model produces an overestimation of the respective running velocities.

(c) Impact factor and contact time

The impact factor  $k_{p,j}$  is defined as:

$$k_{p,j} = \frac{\pi}{2t_{c,rel,j}} \quad (4)$$

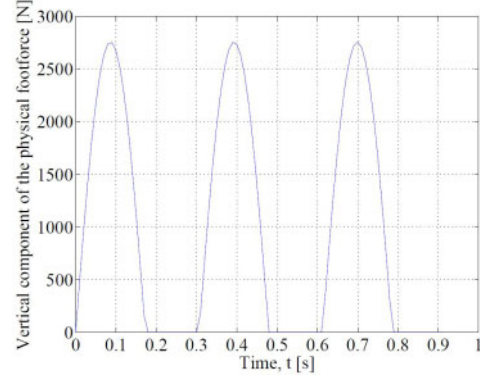


Figure 2. Physical foot-force due to one single runner during the 1s of Monte Carlo simulation.

where  $t_{c,rel,j}$  is the relative contact time, i.e. the ratio of the contact time  $t_{p,j}$  to the period  $T_{p,j}$ .

Kasperski and Sahanci (2008) observed how the relative contact time,  $t_{c,rel}$ , decreases with the running speed. A linear fit to the observed negative trend, can be written as:

$$t_{c,rel,j} = -0.07 v_{p,j} + 0.95 \quad (5)$$

According to the linear model, for the experienced running speeds, the contact time  $t_c$  represents a fraction of the period  $T_p$ , and the ratio of the two parameters varies between 0.5 to 0.9.

Figure 2 shows the physical foot-force due to a single pedestrian, running at a mean frequency of 2.5 Hz, during 1 s (first three cycles) of simulation. The force amplitude is about three times the selfweight of the runner, in agreement with previous descriptions (Bachmann and Amman, 1987).

## 2.2 The reference footbridge

The footbridge response is evaluated assuming the reference footbridge configuration of Table 1. Furthermore, the structure is assumed linear, viscously damped, and with well separated mode shapes. For each mode of vibration the response can be found solving the ordinary second order differential equation of motion is solved:

$$\ddot{q}_j(t) + 2\zeta_j \omega_j \dot{q}_j(t) + \omega_j^2 q_j(t) = p_j(t) \quad (6)$$

where  $\zeta_j$ ,  $\omega_j$ , and  $p_j(t)$  are the modal damping ratio, the angular frequency and the normalized modal load for the  $j$ th vibration mode respectively.

## 3 REFERENCE RESPONSE SPECTRUM

The reference response spectrum, defined as the peak acceleration associated with a certain return period, has been created based on Monte Carlo simulations. For each simulation the footbridge response is divided

Table 1. Reference footbridge configuration.

Parameter	Value
Span length, $L$ [m]	50
Modal mass, $M$ [kg]	100000
Damping ratio, $\zeta$ [–]	0.005
Mode shape, $\Phi(x)$	$\sin(\pi x/L)$
Flow rate, $\lambda$ [ped/s]	1

into  $N$  non-overlapping time windows, each of duration  $T_w$ ; from each time window the peak acceleration is then extracted and a General Extreme Value (GEV) distribution is fitted to the peaks. The cumulative distribution function of the chosen distribution is:

$$H(x) = \exp\left\{-\left[1 + \xi \frac{x - \mu}{\psi}\right]^{-1/\xi}\right\} \quad (7)$$

The function is defined in the region where  $1 + \xi(x - \mu)/\psi > 0$ , else  $H(x) = 1$  or 0. The parameters  $\xi$ ,  $\mu$ , and  $\psi$  are the shape, location and scale parameters respectively. The peak acceleration, as a function of the return period  $T_R$ , is expressed as:

$$\hat{a}(T_R) = \mu - \frac{\psi}{\xi} \left\{ 1 - \left[ -\ln\left(1 - \frac{1}{T_R}\right) \right]^{T_w} \right\}^{-1/\xi} \quad (8)$$

The parameters have been determined on the basis of simulations conducted at 53 distinct frequency ratios,  $r$ , in the range of 0.48 to 1.52. Each simulation is conducted for a total time of 8 hours or,  $T_{tot} = 28800$  s. The response, minus the first and last 200 cycles, is divided into 96 time windows, each lasting 300 s. A time window of 300 s is chosen, as this length is much longer than the time required by each pedestrian to cross the bridge, i.e. each pedestrian will only ideally affect results in one time window.

The GEV parameters as a function of the frequency ratio  $r$  are proposed, so that:

$$\mu_{GEV}(r) = A_\mu \exp\left\{-\frac{(D_\mu - r)^2}{B_\mu}\right\} + C_\mu \quad (9)$$

$$\psi_{GEV}(r) = A_\psi \exp\left\{-\frac{(D_\psi - r)^2}{B_\psi}\right\} + C_\psi \quad (10)$$

$$\xi_{GEV}(r) = \bar{\xi} = \text{mean}[\xi(r)] \quad (11)$$

The return period reference response spectrum is then obtained after substitution of equations (9) to (11) into equation (8), so that:

$$\hat{a}(T_R) = A_\mu \exp\left\{-\frac{(D_\mu - r)^2}{B_\mu}\right\} + C_1 \exp\left\{-\frac{(D_\psi - r)^2}{B_\psi}\right\} + C_2 \quad (12)$$

Table 2. Coefficients for the GEV distribution.  $T_R = 600$  s.

$f_p$ [Hz]	2.5	3.2
$A_\mu$	1.6360	1.7221
$B_\mu$	0.0127	0.0133
$C_\mu$	0.0976	0.1278
$D_\mu$	1.0000	1.0000
$A_\psi$	0.2393	0.2754
$B_\psi$	0.0185	0.0160
$C_\psi$	0.0091	0.0129
$D_\psi$	1.0034	1.0000
$\bar{\xi}$	-0.0634	-0.0231
$C_1$	0.1622	0.4993
$C_2$	0.1038	0.1512

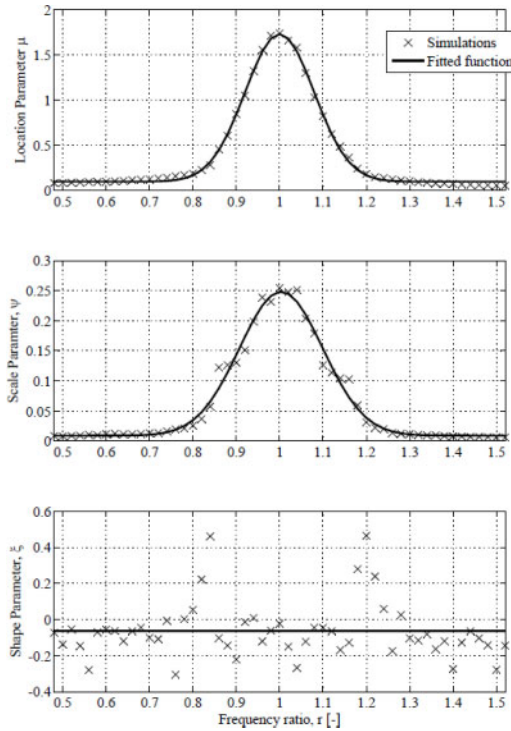


Figure 3. Location, scale and shape parameters from simulations shown with the fitted function.

The coefficients in equations (9) to (12) are presented in Table 2, as obtained for two selected populations, characterized by mean pacing frequencies, i.e.  $f_p = 2.5$  Hz and 3.2 Hz.

Figure 3 shows the location, scale and shape parameters as obtained from the simulations, as well as the fitted functions.

Figure 4 shows the reference acceleration response spectra, as obtained using equation (12), for two different return periods, namely  $T_R = 600$  s and  $T_R = 10^6$  s. The blue crosses represent the peak accelerations of each time window.

The reference response spectra, obtained for two running pedestrian populations, with characteristic

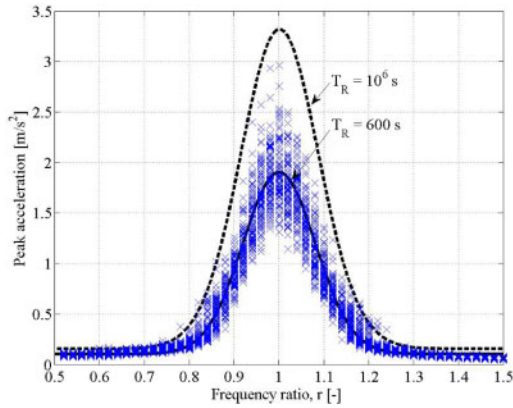


Figure 4. Acceleration response spectra for  $T_R = 600$  s and  $T_R = 10^6$  s using smoothened parameters (black lines). The blue dots represent the actual time window peak accelerations from the simulations.

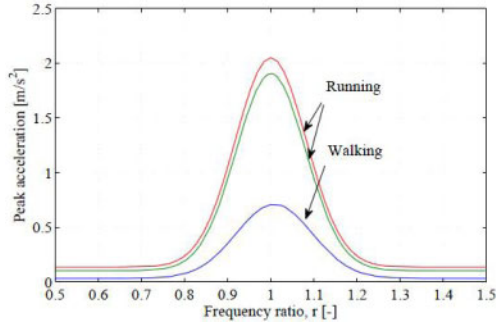
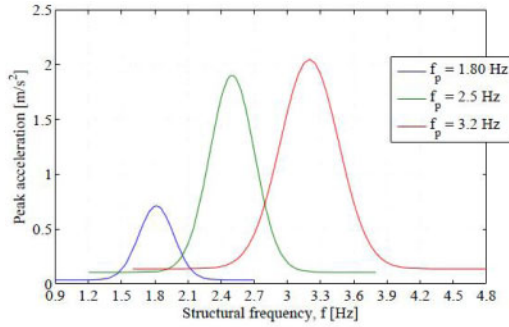


Figure 5. Reference response spectra for 10 minutes peak acceleration for three different pedestrian populations.

mean pacing frequencies of 2.5 (slow running) and 3.2 Hz (fast running) can be seen in Figure 5.

The spectra are plotted together with the spectrum developed by Georgakis and Ingólfsson (2008) for a walking pedestrian populations characterized by a mean pacing frequency of 1.80 Hz.

As part of the generalized Response Spectrum Method, correction factors are introduced by Georgakis & Ingólfsson (2008) to calculate the response in structural as well as crowd configurations differing from the reference bridge. Numerical simulations have confirmed their applicability, in case of running.

The response spectra, as presented, allow for the determination of the peak acceleration associated with a probability of exceedance equal to 50%, for the specified return period. For design purposes it might be necessary to predict the peak accelerations associated with lower probabilities of exceedance, e.g. 10%. Therefore, based on simulations, a further correction factor,  $RE$ , is proposed which allows for the quantification of the peak acceleration associated with a random probability of exceedance, ranging from 0 to 100%, so that:

$$R_E = 0.12 \sin(7p_x + 0.22) + 0.77 p_x + 0.65 \quad (13)$$

where the probability of exceedance,  $p_x$ , is expressed as a number between 0 and 1.

#### 4 CONCLUSIONS

An evaluation of the response of a footbridge subjected to the loading of a crowd when running is developed on the basis of the already validated Response Spectrum Method as described by Georgakis and Ingólfsson (2008). The method allows for a determination of the return period dependant modal peak acceleration for an arbitrary bridge and crowd.

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